A spatial extremes characterization of the annual maxima precipitation in Madeira Island

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Abstract

A variety of statistical tools such as copulas and spatial max-stable processes have been used in the most recent decades for modelling spatial extreme data. Our aim is to give a spatial extremes characterization of Madeira Island's annual maxima precipitation using annual maximum daily precipitation data from 17 rain gauge stations throughout the island.

Key Words: Statistics of extremes, spatial extremes, copula functions, extreme precipitation.

1 Introduction

Extreme rainfall events have triggered a significant number of flash floods, landslides and debris flows in Madeira Island, a volcanic island located in the north-east Atlantic Ocean between latitudes 32°30'N–33°30'N and longitudes 16°30'W–17°30'W, along its past and recent history. One of the most significant events was the one that happened on the 20th of February 2010, which caused 45 casualties, six missed people and extensive damage to properties and infrastructures [3, 5].

The spatial distribution of precipitation in Madeira Island is strongly affected by its highly rugged topography and our aim is to give a spatial extremes characterization of Madeira's annual maxima precipitation through a copula function [6], using annual maximum daily precipitation data from 17 rain gauge stations spread throughout the island provided for this study by the Department of Hydraulics and Energy Technologies of the Madeira Regional Laboratory of Civil Engineering.

A review of spatial extremes methods based on latent variables, copulas and spatial max-stable processes is presented by Davison et al. [2], which refer that appropriately chosen copula or max-stable models seem to be essential for the spatial modelling of extremes. The importance of max-stable and copula approaches for modelling spatial dependence is also emphasised by other authors such as Cooley et al. [1]. Although the present work and most of the studies on spatial extremes mentioned in the reviews cited above focus on modelling block maxima data, it is important to mention here that there are other studies (e.g., [7]) where these same methods are also applied to continuous time series of daily aggregated precipitation.

2 Statistical Analysis

2.1 Data

The data used in this study correspond to the highest values of annual daily precipitation on the island of Madeira, in the period of 1959–1980. The data come from 17 rain gauge stations maintained in the past by the General Council of the Autonomous District of Funchal, whose approximate location is indicated in Figure 1 by the letters A to Q, grouped according to the proximity of the stations to each other. Table 1 provides information about each of these rain gauge stations, namely their identification, geographical location and altitude.

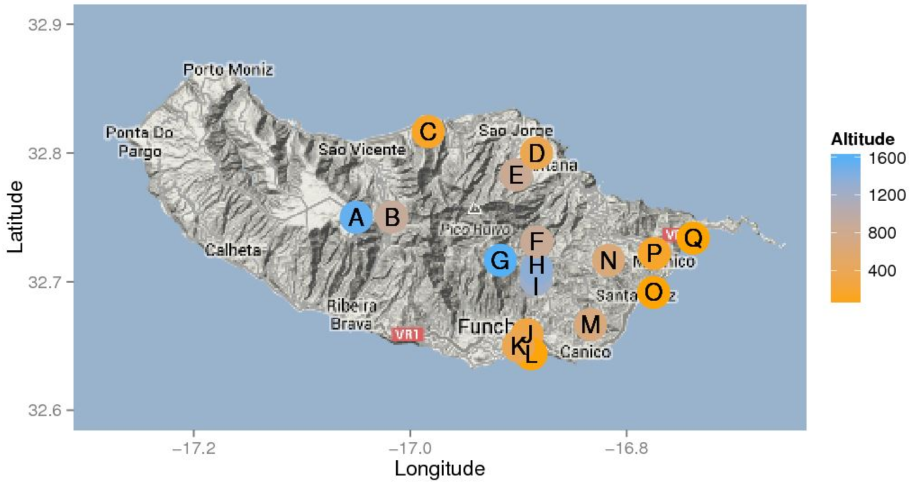

Figure 1: Location and altitude of the rain gauge stations considered in this study.

Table 1: Details of the rain gauge stations used in this study.

Station Name/ ID	Latitude	Longitude	Altitude
Bica da Cana (A)	32°45'N	17°03'W	1560 m
Encumeada (B)	32°45'N	17°01'W	900 m
Ponta Delgada (C)	32°49'N	16°59'W	136 m
Santana (D)	32°48'N	16°53'W	380 m
Queimadas (E)	32°46'N	16°54'W	860 m
Ribeiro Frio (F)	32°43'N	16°53'W	874 m
Areeiro (G)	32°43'N	16°55'W	1610 m
Poiso (H)	32°42'N	16°53'W	1360 m
Montado do Pereiro (I)	32°42'N	16°53'W	1260 m
Bom Sucesso (J)	32°39'N	16°54'W	290 m
Sanatório (K)	32°39'N	16°54'W	380 m
Funchal (L)	32°38'N	16°53'W	58 m
Camacha (M)	32°40'N	16°50'W	680 m
Santo da Serra (N)	32°43'N	16°49'W	660 m
Santa Catarina (O)	32°41'N	16°46'W	49 m
Machico (P)	32°43'N	16°47'W	160 m
Caniçal (Q)	32°44'N	16°44'W	40 m

2.2 Methodology

Let $\mathbb{I} = [0, 1]$ and $u = (u_1, ..., u_d) \in \mathbb{I}^d$, with $d \geq 3$. A d-copula C satisfying the relationship $C(u_1^t, ..., u_d^t) = C^t(u_1, ..., u_d)$ for all $t > 0$ is called an Extreme Value (EV) copula [6].

It is known that if A and B are d–copulas then

$$C_{\alpha_1,...,\alpha_d}(u) = A\left(u_1^{\alpha_1}, ..., u_d^{\alpha_d}\right) B\left(u_1^{1-\alpha_1}, ..., u_d^{1-\alpha_d}\right)$$

defines a family of d-copulas $C_{\alpha_1,...,\alpha_d}$ with parameters $\alpha_1, ..., \alpha_d \in \mathbb{I}$, and that if A and B are EV copulas, then the copula C is also an EV copula. Besides, in this way there is an increase in the number of parameters, which improves the description of the dependence.

So, choosing $d = 3$, the three–dimensional Cuadras–Augé copula A, and the product copula B, defined respectively by

$$A(u_1, u_2, u_3) = (u_1 u_2 u_3)^{1-\theta} \left[\min(u_1, u_2, u_3)\right]^{\theta}, \theta \in \mathbb{I},$$

and

$$B(u_1, u_2, u_3) = u_1 u_2 u_3,$$

we have the following family of EV copulas

$$C^*_\beta(u_1, u_2, u_3) = \Pi_{i=1}^3 u_i^{1-\beta_i} \min\left(u_1^{\beta_1}, u_2^{\beta_2}, u_3^{\beta_3}\right),$$

where $\beta_i = \theta\alpha_i \in \mathbb{I}$.

We notice that the two-dimensional marginals of C^* are Marshall-Olkin bivariate copulas with parameters (β_i, β_j) for $i \neq j$, with $i, j = 1, 2, 3$, and therefore

$$\tau^{i,j} = \frac{1}{1/\beta_i + 1/\beta_j - 1}.$$

This fact allows the calculation of the parameters β_1, β_2 and β_3 using the estimated values of the Kendall's τ measure of association by

$$\frac{1}{\beta_i} = \frac{1}{2}\left(1 + \frac{1}{\tau^{i,j}} + \frac{1}{\tau^{i,k}} - \frac{1}{\tau^{j,k}}\right),$$

where (i, j, k) is a permutation of $(1, 2, 3)$.

Of practical interest is the trivariate event given by

$$E_q = \{X_1 > x_{1,q}, X_2 > x_{2,q}, X_3 > x_{3,q}\},$$

where X_i denotes the observation at the i–th rain gauge station, and $x_{i,q}$ is the q-order quantile of X_i with $q \in (0,1)$. The return period r_q of E_q is given by $r_q = \frac{1}{p_q}$, with $p_q = \mathbb{P}(E_q)$. It is assumed that the X_i's are joined via the EV copula C^*. The probability p_q is calculated by

$$p_q = 1-u_1-u_2-u_3-C^*_\beta(u_1,u_2,u_3)+C^*_\beta(u_1,u_2,1)+C^*_\beta(u_1,1,u_3)+C^*_\beta(1,u_2,u_3),$$

with $u_1 = u_2 = u_3 = q$ [6].

2.3 Results

In a previous work we obtained the Kendall's τ estimates for the pairs of stations listed in Table 1. Table 2 shows the pairs, of the northern stations of Madeira Island, for which the independence is rejected at the significance level $\alpha = 0.05$ by the application of a test of independence based on the empirical version of the Kendall's τ association measure [4].

Table 2: Kendall's τ estimates.

Station 1	Station 2	$\widehat{\tau}$
Bica da Cana (A)	Encumeada (B)	0.47
Bica da Cana (A)	Queimadas (E)	0.45
Encumeada (B)	Ponta Delgada (C)	0.31
Encumeada (B)	Santana (D)	0.31
Encumeada (B)	Queimadas (E)	0.43
Ponta Delgada (C)	Santana (D)	0.31
Ponta Delgada (C)	Queimadas (E)	0.55
Santana (D)	Queimadas (E)	0.36

The location and altitude range of the northern rain gauge stations are showed in Figure 2. All the corresponding groups created by two pairs of associated stations are described in Table 3. The values for the parameters β_i, with $i \in \{1,2,3\}$, for the four groups obtained are also presented in Table 3.

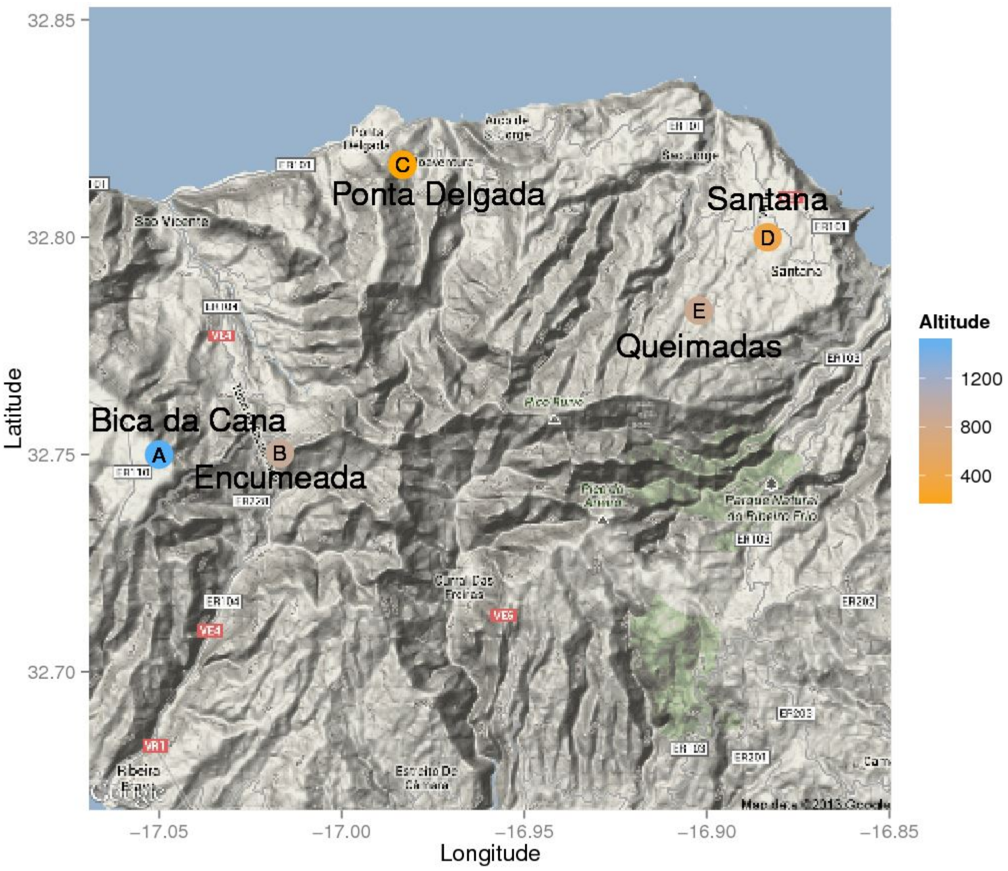

Figure 2: Location and altitude range of the rain gauge stations in the north of the island.

Table 3: Northern groups and corresponding parameters β_1, β_2 and β_3.

Group	Station 1	Station 2	Station 3	β_1	β_2	β_3
1	A	B	E	0.6613	0.6190	0.5848
2	B	D	E	0.5300	0.4275	0.6950
3	B	C	D	0.4733	0.4733	0.4733
4	C	D	E	0.6123	0.3857	0.8438

The calculated return periods showed lower values for the group in which are included the two stations with higher altitude, namely the Group 1 (see Table 4). The highest return periods correspond to the stations of Group 4, constituted by the three stations with the lowest altitude of the northern side of Madeira Island.

Table 4: Return periods in years, $r_{0.98}$ and $r_{0.99}$, and associated probabilities for the northern groups.

Group	$p_{0.98}$	$r_{0.98}$	$p_{0.99}$	$r_{0.99}$
1	0.01175	85.10	0.00586	170.60
2	0.00863	115.95	0.00429	232.92
3	0.00952	105.07	0.00475	210.72
4	0.00782	127.90	0.00388	257.54

The pairs, formed by the southern stations of Madeira Island, for which the independence is rejected at the significance level $\alpha = 0.05$ by the application of a test of independence based on the empirical version of the Kendall's τ association measure [4], are presented in Table 5.

Table 5: Kendall's τ estimates.

Station 1	Station 2	$\widehat{\tau}$
Ribeiro Frio (F)	Poiso (H)	0.44
Ribeiro Frio (F)	Montado do Pereiro (I)	0.40
Ribeiro Frio (F)	Camacha (M)	0.32
Ribeiro Frio (F)	Santo da Serra (N)	0.31
Areeiro (G)	Poiso (H)	0.45
Areeiro (G)	Camacha (M)	0.35
Poiso (H)	Montado do Pereiro (I)	0.43
Poiso (H)	Camacha (M)	0.39
Poiso (H)	Santo da Serra (N)	0.31
Montado do Pereiro (I)	Camacha (M)	0.42
Montado do Pereiro (I)	Santo da Serra (N)	0.38
Bom Sucesso (J)	Funchal (L)	0.50
Bom Sucesso (J)	Sanatório (K)	0.34
Bom Sucesso (J)	Santa Catarina (O)	0.32
Sanatório (K)	Santo da Serra (N)	0.46
Sanatório (K)	Santa Catarina (O)	0.39
Sanatório (K)	Machico (P)	0.39
Sanatório (K)	Caniçal (Q)	0.45
Funchal (L)	Santa Catarina (O)	0.37
Camacha (M)	Santo da Serra (N)	0.47
Santo da Serra (N)	Caniçal (Q)	0.51
Santa Catarina (O)	Caniçal (Q)	0.48
Machico (P)	Caniçal (Q)	0.32

On the southern side of the island were obtained 16 groups of three stations associated two by two. The descrition of these groups formed with the southern rain gauge stations, whose location and altitude range are indicated in Figure 3, is presented in Table 6.

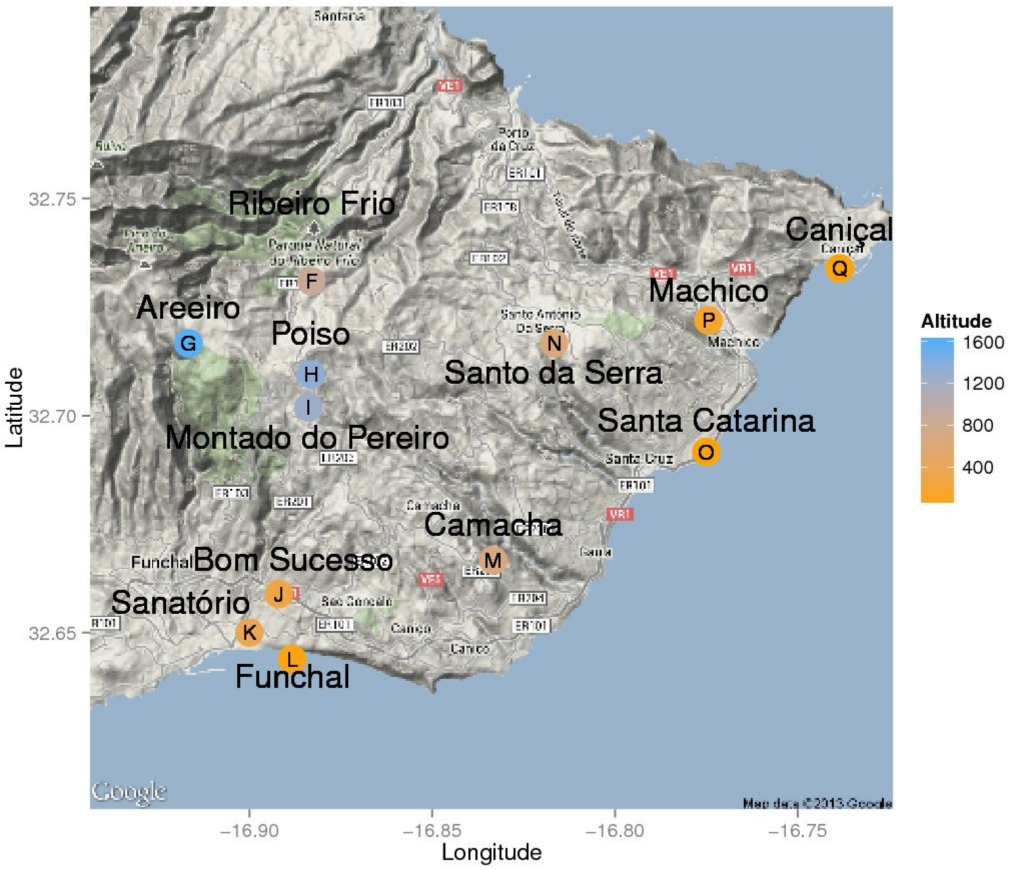

Figure 3: Location and altitude range of the rain gauge stations in the south of the island.

The copula C^*_β for each one of the southern station groups is defined by the parameters β_1, β_2 and β_3, whose values are also displayed in Table 6. The probabilities and the corresponding return periods for the southern station groups are presented in Table 7. The lowest and highest return periods were obtained for the stations of Groups 16 and 9, respectively.

Table 6: Southern groups and corresponding parameters β_1, β_2 and β_3.

Group	Station 1	Station 2	Station 3	β_1	β_2	β_3
1	G	H	M	0.5359	0.6357	0.5022
2	F	H	I	0.5802	0.6455	0.5629
3	F	I	M	0.4712	0.7257	0.4993
4	F	I	N	0.4885	0.6883	0.4590
5	F	H	M	0.5217	0.7375	0.4529
6	F	H	N	0.6111	0.6111	0.3862
7	H	I	M	0.5700	0.6364	0.5526
8	H	I	N	0.5102	0.7322	0.4413
9	F	M	N	0.3829	0.6608	0.6195
10	H	M	N	0.4290	0.8110	0.5278
11	I	M	N	0.5148	0.6952	0.5920
12	J	L	O	0.5844	0.7759	0.4143
13	J	K	O	0.4442	0.5917	0.5336
14	K	O	Q	0.5401	0.5839	0.7295
15	K	P	Q	0.7151	0.4477	0.5287
16	K	N	Q	0.5822	0.6867	0.6647

The lowest and highest return periods were obtained for the stations of Groups 16 and 9, respectively. The only rain gauge station common to both groups is Santo da Serra (N). The other two stations in Group 9 have higher altitude than Santo da Serra (N), but the opposite occurs in Group 16. In addition to Group 16, there are two groups with similar return periods, namely Groups 2 and 7. Both groups are formed by two stations with altitudes higher than 1000m and one station with an altitude close to 670m.

There are also the Groups 14, 11 and 1, with $92 \leq r_{0.98} \leq 100$ and $184 \leq r_{0.99} \leq 200$. All the other groups have the return periods $r_{0.98}$ and $r_{0.99}$ higher than 105 and 210, respectively. The return periods increase until $r_{0.98} \approx 116$ and $r_{0.99} \approx 233$ for Groups 3, 4, 5, 15, 13, 8 and 10. For Groups 12, 6 and 9, it is observed that $119 \leq r_{0.98} \leq 129$ and $240 \leq r_{0.99} \leq 260$.

All the results presented were obtained by the application of the Rcmdr R package [8]. The maps were created by the ggmap R package [8].

Table 7: Return periods in years, $r_{0.98}$ and $r_{0.99}$, and associated probabilities for the southern groups.

Group	$p_{0.98}$	$r_{0.98}$	$p_{0.99}$	$r_{0.99}$
1	0.01010	98.99	0.00504	198.56
2	0.01131	88.41	0.00564	177.24
3	0.00948	105.47	0.00472	211.59
4	0.00923	108.25	0.00460	217.19
5	0.00912	109.60	0.00454	220.01
6	0.00782	127.74	0.00389	257.21
7	0.01111	90.04	0.00554	180.53
8	0.00889	112.46	0.00443	225.77
9	0.00777	128.78	0.00386	259.36
10	0.00865	115.56	0.00430	232.12
11	0.01036	96.51	0.00516	193.63
12	0.00838	119.39	0.00417	240.08
13	0.00895	111.67	0.00446	224.24
14	0.01086	92.07	0.00542	184.65
15	0.00902	110.83	0.00449	222.52
16	0.01171	85.42	0.00584	171.30

3 Conclusion

A variety of statistical tools, such as copulas and spatial max-stable processes, have been used in the most recent decades for modelling spatial extreme data. In this work, an application of copula functions to the annual 1-day maximum precipitation data from 17 rain gauge stations in Madeira Island is presented.

In a previous work we found the Kendall's τ estimates and tested independence for pairs of stations by a test based on the empirical version of this association measure. In this study, the mentioned pairs of stations were used to form 20 groups of three stations associated two by two. Four of these groups are situated on the northern side and the remaining are located on the southern side of Madeira Island. The values for the parameters β_i, with $i \in \{1, 2, 3\}$, that define the copula C_β^* were determined for each one of the groups.

The event $E_q = \{X_1 > x_{1,q}, X_2 > x_{2,q}, X_3 > x_{3,q}\}$, where X_i denotes the observation at the i–th rain gauge station and $x_{i,q}$ is the q-order quantile of the variable X_i, with $q \in (0,1)$, is an event of practical interest. For $q = 0.98$ and $q = 0.99$, the probability p_q and the corresponding return period r_q were calculated for each one of the northern and southern groups. Both of these sets of station groups can be ordered according to the values of the return periods. None of the natural factors that have greater influence on the precipitation values, such as altitude and proximity to the sea, stand alone in the spatial characterization of extreme precipitation in Madeira Island, which can be explained by the extremely complex topography of the island.

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